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Mass- and charge-renormalizations in quantum electrodynamics without use of the interaction representation

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Summary

The differential equations of quantum electrodynamics are solved in a few special cases without use of the interaction representation of TOMONAGA and SCHWINGER. The results agree with those obtained earlier and the calculations are, at least in the problems treated here, somewhat simpler than before. It may thus be pointed out that the mass- and charge-renormalizations can be carried out without any reference to a system of space-like surfaces. This is especially convenient when particles of integer spin are present, as the interaction operator of SCHWINGER then contains the normal unit vectors of these surfaces. The end formulae, however, must be independent of the normals.

I. Introduction

During the last years considerable progress in the field of quantum electrodynamics has been made due to the conceptions of mass- and charge-renormalizations. With the use of these ideas it has been possible to separate the divergent parts of cross-sections, energy-level shifts and so on, and to extract physically significant results from the remaining terms. As far as those formulae have been compared with experiments the agreement is good. In all the calculations referred to above, the differential equations of quantum electrodynamics have been solved in two steps. The electromagnetic and the electronic field operators, which are originally given in a Heisenberg representation, are first transformed to an intermediate representation (the interaction representation) where the field equations are the same as those for the free fields.[1] The remaining time dependence is taken over by the state vector $|\psi\rangle$, the variation of which is given by

$$i\frac{\delta|\psi\rangle}{\delta\sigma(x)} = H(x)|\psi\rangle. \tag{1}$$

The solution of equation (1) is the second step in the calculation. The usual procedure is to transform the state vector by a series of canonical transformations, until the right-hand side of the equation of motion is of a sufficiently small order of magnitude and can be neglected. The new state vector is then a constant — that is, one has returned to the original Heisenberg representation. An arbitrary operator F is then given as a power series in the coupling constant e

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[1] J. SCHWINGER: Phys. Rev. *74*, 1439 (1948).

$$F = F^{(0)} + e\,F^{(1)} + e^2\,F^{(2)} + \cdots \tag{2}$$

where the $F^{(i)}$ are built up of commutators and ε-functions or of P-symbols.[2]

It might be of interest to investigate, if it is possible to treat the same equations without using the interaction representation. In the calculations below we will retain the Heisenberg representation and substitute the expansions (2) directly into the original equations.[3] The coefficients of different powers of e will then be put equal to zero, and the formulae obtained in this way will be shown to be identical with those of SCHWINGER.

II. Vacuum polarization in an external field for particles of spin $\frac{1}{2}$

We begin with this simple problem where the only operator equation is

$$\left(\gamma\frac{\partial}{\partial x} + m\right)\psi(x) = i\,e\,\gamma\,A(x)\,\psi(x). \tag{3}$$

The potentials $A_\nu(x)$ are here given functions of space and time and no operators. We put

$$\psi(x) = \psi^{(0)}(x) + e\,\psi^{(1)}(x) + \cdots \tag{4}$$

and obtain from (3)

$$\left(\gamma\frac{\partial}{\partial x} + m\right)\psi^{(n+1)}(x) = i\,\gamma\,A(x)\,\psi^{(n)}(x) \tag{5 a}$$

$$\left(\gamma\frac{\partial}{\partial x} + m\right)\psi^{(0)}(x) = 0. \tag{5 b}$$

The operators $\psi^{(0)}(x)$ here correspond to the operators $\psi(x)$ of SCHWINGER and the same argument as used by him will give the formulae[1, 4]

$$\{\bar\psi^{(0)}_\alpha(x),\,\psi^{(0)}_\beta(x')\} = -\,i\,S_{\beta\alpha}(x'-x) \tag{6 a}$$

$$\langle\,[\bar\psi^{(0)}_\alpha(x),\,\psi^{(0)}_\beta(x')]\,\rangle_0 = S^{(1)}_{\beta\alpha}(x'-x). \tag{6 b}$$

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[2] F. J. DYSON: Phys. Rev. *75*, 486 (1949).

[3] A similar formalism has been developed independently by C. N. YANG, Princeton, and I should like to take this opportunity of thanking him for sending me his unpublished results.

[4] J. SCHWINGER: Phys. Rev. *75*, 651 (1949).

As boundary conditions for the equation (5 a) we use ($n \neq 0$)

$$\psi^{(n)}(x) \to 0 \quad \text{for} \quad x_0 \to -\infty \tag{7 a}$$

$$\psi^{(n)}(x) \mid \psi \rangle \to 0 \quad \text{for} \quad x_0 \to +\infty. \tag{7 b}$$

The last equation expresses the absence of real effects and is, in fact, a condition for the given external field. Equation (7 a) is then the important boundary condition for the operators. With the aid of the formulae

$$\left(\gamma \frac{\partial}{\partial x} + m\right) S(x) = 0 \tag{8 a}$$

$$\left(\gamma \frac{\partial}{\partial x} + m\right) \bar{S}(x) = -\delta(x) \tag{8 b}$$

we now obtain from (5 a) a recursion formula for $\psi^{(n)}(x)$

$$\psi^{(n+1)}(x) = -i \int S_R(x - x')\, \gamma A(x')\, \psi^{(n)}(x')\, dx' \tag{9 a}$$

$$S_R(x) = \bar{S}(x) - \tfrac{1}{2} S(x). \tag{9 b}$$

From (9 b) it is immediately seen that (7 a) is fulfilled. Equation (7 b) gives

$$\int S(x - x')\, \gamma A(x')\, \psi^{(n)}(x')\, dx' \mid \psi \rangle = 0 \tag{10}$$

and hence we always get the correct *expectation values* if we use

$$\psi^{(n+1)}(x) = -i \int \bar{S}(x - x')\, \gamma A(x')\, \psi^{(n)}(x')\, dx' \tag{9 c}$$

as the solution of (5 a). From (9 c) we obtain without difficulty

$$\psi^{(n)}(x) = (-i)^n \int \cdots \int dx' \cdots dx^n\, \bar{S}(x - x')\, \gamma A(x')\, \bar{S}(x' - x'') \cdot$$
$$\cdot\, \gamma A(x'') \cdots \bar{S}(x^{n-1} - x^n)\, \gamma A(x^n)\, \psi^{(0)}(x^n). \tag{11}$$

Writing the expectation value of the current operator as

$$\langle j_\mu(x) \rangle_0 = \sum_{n=0}^{\infty} e^{n+1} \langle j_\mu^{(n)}(x) \rangle_0 \tag{12}$$

we get

$$\langle j_\mu^{(n)}(x) \rangle_0 = \frac{(-i)^{n-1}}{2} \sum_{m=0}^{n} \int \cdots \int dx' \cdots dx^n\, Sp\, [\gamma_\mu \bar{S}(x - x')\, \gamma A(x') \cdots$$
$$\cdots \gamma A(x^m)\, S^{(1)}(x^m - x^{m+1})\, \gamma A(x^{m+1}) \cdots \gamma A(x^n)\, \bar{S}(x^n - x)]. \tag{13}$$

This formula has earlier been obtained by the author using the original formulation of SCHWINGER.[5]

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[5] G. KÄLLÉN: Helv. Phys. Acta *22*, 637 (1949).

III. Vacuum polarization in an external field for particles of spin 0

In this case the differential equation for the operator is

$$(\Box - m^2)\,\varphi(x) = i\,e\left[A_\nu(x)\,\frac{\partial\varphi(x)}{\partial x_\nu} + \frac{\partial}{\partial x_\nu}\left(A_\nu(x)\,\varphi(x)\right)\right] + e^2 A_\nu(x)\,A_\nu(x)\,\varphi(x). \tag{14}$$

Putting

$$\varphi(x) = \sum e^n\,\varphi^{(n)}(x) \tag{15}$$

as before, we get

$$\begin{aligned}\varphi^{(n+1)}(x) = &-i\int A_\nu(x')\left[\overline{\triangle}(x-x')\,\frac{\partial}{\partial x'_\nu} + \frac{\partial\overline{\triangle}(x-x')}{\partial x_\nu}\right]\varphi^{(n)}(x')\,dx' + \\ &+ (-i)^2\iint A_{\nu_1}(x')\,A_{\nu_2}(x'')\,\overline{\triangle}(x-x')\,\delta_{\nu_1\nu_2}\,\delta(x'-x'')\,\varphi^{(n-1)}(x'')\,dx'\,dx''\end{aligned} \tag{16}$$

and hence

$$\begin{aligned}\varphi^{(n)}(x) = (-i)^n\int\cdots\int dx'\cdots dx^n\,A_{\nu_1}(x')\cdots A_{\nu_n}(x^n)\Bigg\{&\left[\overline{\triangle}(x-x')\,\frac{\partial}{\partial x'_{\nu_1}} + \right.\\ &\left.+\frac{\partial\overline{\triangle}(x-x')}{\partial x_{\nu_1}}\right]\left[\overline{\triangle}(x'-x'')\,\frac{\partial}{\partial x''_{\nu_2}} + \frac{\partial\overline{\triangle}(x'-x'')}{\partial x'_{\nu_2}}\right]\cdots\left[\overline{\triangle}(x^{n-1}-x^n)\,\frac{\partial}{\partial x^n_{\nu_n}} + \right.\\ &\left.+\frac{\partial\overline{\triangle}(x^{n-1}-x^n)}{\partial x^{n-1}_{\nu_n}}\right] + \sum_{i=1}^{n-1}\left[\overline{\triangle}(x-x')\,\frac{\partial}{\partial x'_{\nu_1}} + \frac{\partial\overline{\triangle}(x-x')}{\partial x_{\nu_1}}\right]\cdots\\ &\cdots\left[\overline{\triangle}(x^{i-2}-x^{i-1})\,\frac{\partial}{\partial x^{i-1}_{\nu_{i-1}}} + \frac{\partial\overline{\triangle}(x^{i-2}-x^{i-1})}{\partial x^{i-2}_{\nu_{i-1}}}\right]\overline{\triangle}(x^{i-1}-x^i)\,\delta_{\nu_i\nu_{i+1}}\,\delta(x^i-x^{i+1})\,\cdot\\ &\cdot\left[\overline{\triangle}(x^{i+1}-x^{i+2})\,\frac{\partial}{\partial x^{i+2}_{\nu_{i+2}}} + \frac{\partial\overline{\triangle}(x^{i+1}-x^{i+2})}{\partial x^{i+1}_{\nu_{i+2}}}\right]\cdots\\ &\cdots\left[\overline{\triangle}(x^{n-1}-x^n)\,\frac{\partial}{\partial x^n_{\nu_n}} + \frac{\partial\overline{\triangle}(x^{n-1}-x^n)}{\partial x^{n-1}_{\nu_n}}\right] + \\ &+\sum_{i<j}\cdots\overline{\triangle}(x^{i-1}-x^i)\,\delta_{\nu_i\nu_{i+1}}\,\delta(x^i-x^{i+1})\cdots\overline{\triangle}(x^{j-1}-x^j)\,\delta_{\nu_j\nu_{j+1}}\,\delta(x^j-x^{j+1})\cdots\\ &+\sum_{i<j<k}\cdots+\cdots\Bigg\}\,\varphi^{(0)}(x^n).\end{aligned} \tag{17}$$

Rearranging the terms in the hermitian conjugate of (17) according to the scheme

$$\left(f(x)\,\frac{d}{dx} - \frac{d\,f(x)}{dx}\right)g(x) = -\left(g(x)\,\frac{d}{dx} - \frac{d\,g(x)}{dx}\right)f(x)$$

we obtain

$$\varphi^{(n)*}(x) = (-i)^n \int \cdots \int d x' \cdots d x^n A_{\nu_n}(x^n) \cdots A_{\nu_1}(x') \left\{ \left[\varphi^{(0)*}(x^n) \frac{\partial}{\partial x^n_{\nu_n}} - \right. \right.$$

$$\left. - \frac{\partial \varphi^{(0)*}(x^n)}{\partial x^n_{\nu_n}} \right] \left[\overline{\triangle}(x^n - x^{n-1}) \frac{\partial}{\partial x^{n-1}_{\nu_{n-1}}} + \frac{\partial \overline{\triangle}(x^n - x^{n-1})}{\partial x^n_{\nu_{n-1}}} \right] \cdots$$

$$\left. \cdots \left[\overline{\triangle}(x'' - x') \frac{\partial}{\partial x'_{\nu_1}} + \frac{\partial \overline{\triangle}(x'' - x')}{\partial x''_{\nu_1}} \right] \overline{\triangle}(x' - x) + \sum_i \cdots + \cdots \right\}. \tag{18}$$

Writing, for convenience, the current operator as

$$j_\mu(x) = \frac{i e}{2} \left(\left\{ \frac{\partial \varphi^*(x)}{\partial x_\mu}, \varphi(x) \right\} - \left\{ \frac{\partial \varphi(x)}{\partial x_\mu}, \varphi^*(x) \right\} \right) -$$

$$- e^2 A_\mu(x) \{\varphi^*(x), \varphi(x)\} = \sum e^{n+1} j^{(n)}_\mu(x) \tag{19}$$

we get in momentum space

$$\langle j^{(n)}_\mu(x) \rangle_0 = \frac{1}{2} \left(\frac{1}{2\pi} \right)^{4n+3} \int \cdots \int d p\, d p' \cdots d p^n\, e^{i x (p - p^n)} A_{\nu_1}(p' - p) \cdots$$

$$\cdots A_{\nu_n}(p^n - p^{n-1}) \cdot P^{(n)}_{\mu \nu_1 \ldots \nu_n} \cdot \left\{ \frac{\delta(p^2 + m^2)}{(p'^2 - p^2) \cdots (p^{n^2} - p^2)} + \cdots \right.$$

$$\left. \cdots + \frac{\delta(p^{n^2} + m^2)}{(p^2 - p^{n^2}) \cdots (p^{n-1^2} - p^{n^2})} \right\} \tag{20}$$

where

$$P^{(n)}_{\mu \nu_1 \ldots \nu_n} = \pi^{(n)} - \sum_i \delta_{\nu_i \nu_{i+1}} (p^{i^2} + m^2) \pi^{(n)}_{i, i+1} + \sum_{i<j} \delta_{\nu_i \nu_{i+1}} \cdot$$

$$\cdot \delta_{\nu_j \nu_{j+1}} (p^{i^2} + m^2)(p^{j^2} + m^2) \pi^{(n)}_{i, i+1, j, j+1} - \sum \cdots + \cdots \tag{21}$$

and

$$\pi^{(n)} = (p'_{\nu_1} + p_{\nu_1})(p''_{\nu_2} + p'_{\nu_2}) \cdots (p^n_{\nu_n} + p^{n-1}_{\nu_n})(p_\mu + p^n_\mu) \tag{22 a}$$

$$\pi^{(n)}_{i, j, \ldots} = \frac{\pi^{(n)}}{(p^i_{\nu_i} + p^{i-1}_{\nu_i})(p^j_{\nu_j} + p^{j-1}_{\nu_j}) \cdots}. \tag{22 b}$$

These formulae have also been obtained earlier with the formalism of SCHWINGER.[5] However, the calculations are here simpler, because we have no trouble with the normals of the space-like surfaces of TOMONAGA and SCHWINGER.

IV. The first radiative corrections to the current operator and the mass-renormalization for particles of spin $\frac{1}{2}$

We now turn to the more complicated problem of the radiative corrections to the current operator for particles of spin $\frac{1}{2}$. In this case both the electron field and the electromagnetic field are operators and we write their equations of motion as

$$\left(\gamma \frac{\partial}{\partial x} + m\right) \psi(x) = \frac{i e}{2} \{A_\nu(x), \gamma_\nu \psi(x)\} + \delta m \, \psi(x) \tag{23}$$

$$\Box A_\mu(x) = -\frac{i e}{2} [\bar{\psi}(x), \gamma_\mu \psi(x)]. \tag{24}$$

The constant m in equation (23) is the experimental mass of the particle and the singular functions, which appear below, must be constructed with this mass. The δm of the right-hand side of (23) must, if possible, be determined in such a way that no divergences appear in the end formulae (except charge-renormalization factors). As before we expand $\psi(x)$, $A_\nu(x)$ and δm in powers of e, and obtain

$$\left(\gamma \frac{\partial}{\partial x} + m\right) \psi^{(n+1)}(x) = \frac{i}{2} \sum_{m=0}^{n} \{A_\mu^{(m)}(x), \gamma_\mu \psi^{(n-m)}(x)\} + \sum_{m=1}^{n+1} \delta m^{(m)} \psi^{(n+1-m)}(x) \tag{25}$$

$$\Box A_\mu^{(n+1)}(x) = -\frac{i}{2} \sum_{m=0}^{n} [\bar{\psi}^{(m)}(x), \gamma_\mu \psi^{(n-m)}(x)]. \tag{26}$$

The first approximations are

$$\psi^{(1)}(x) = -i \int \bar{S}(x-x') \gamma A^{(0)}(x') \psi^{(0)}(x') \, d x' - \delta m^{(1)} \int \bar{S}(x-x') \psi^{(0)}(x') \, d x' \tag{27}$$

$$A_\mu^{(1)}(x) = \frac{i}{2} \int \bar{D}(x-x') [\bar{\psi}^{(0)}(x'), \gamma_\mu \psi^{(0)}(x')] \, d x'. \tag{28}$$

For the corresponding term of the current operator we obtain

$$\begin{aligned} j_\mu^{(1)}(x) = \tfrac{1}{2} \int [\bar{\psi}^{(0)}(x), \gamma_\mu \bar{S}(x-x') \gamma_\nu \psi^{(0)}(x')] A_\nu^{(0)}(x') \, d x' - i \, \delta m^{(1)} \tfrac{1}{2} \cdot \\ \cdot \int [\bar{\psi}^{(0)}(x), \gamma_\mu \bar{S}(x-x') \psi^{(0)}(x')] \, d x' + \text{herm. conj.} \end{aligned} \tag{29}$$

To determine $\delta m^{(1)}$ we compute the one-electron part of (29) and, due to the factor $A_\nu^{(0)}(x')$, get zero for the first term and its hermitian conjugate. Hence

$$\delta m^{(1)} = 0. \tag{30}$$

(As is easily seen from an analogous argument we also obtain, more generally

$$\delta m^{(2n+1)} = 0 \tag{30 a)}$$

In the next approximation we get

$$\psi^{(2)}(x) = -\tfrac{1}{2}\int\!\!\int \bar{S}(x-x')\,\gamma_{\nu_1}\,\bar{S}(x'-x'')\,\gamma_{\nu_2}\,\psi^{(0)}(x'')\,\{A^{(0)}_{\nu_1}(x'),\,A^{(0)}_{\nu_2}(x'')\}\,d\,x'\,d\,x'' +$$
$$+\tfrac{1}{4}\int\!\!\int \bar{S}(x-x')\,\gamma_{\nu_1}\,\{\psi^{(0)}(x')\,[\bar{\psi}^{(0)}(x''),\,\gamma_{\nu_2}\,\psi^{(0)}(x'')]\}\,\delta_{\nu_1\nu_2}\,\bar{D}(x'-x'')\,d\,x'\,d\,x'' -$$
$$-\,\delta\,m^{(2)}\int \bar{S}(x-x')\,\psi^{(0)}(x')\,d\,x'. \qquad (31)$$

As our theory is gauge invariant we can use the formula

$$\langle\{A^{(0)}_{\nu_1}(x'),\,A^{(0)}_{\nu_2}(x'')\}\rangle_0 = \delta_{\nu_1\nu_2}\,D^{(1)}(x'-x'') \qquad (32)$$

and obtain from (31) with the aid of (32)

$$\frac{i}{2}[\bar{\psi}^{(0)}(x),\,\gamma_\mu\,\psi^{(2)}(x)]_{1,0} = -\frac{i}{4}\int\!\!\int[\bar{\psi}^{(0)}(x),\,\gamma_\mu\,\bar{S}(x-x')\,\gamma_{\nu_1}\,\bar{S}(x'-x'')\,\gamma_{\nu_2}\,\psi^{(0)}(x'')]_1\,\cdot$$
$$\cdot\,D^{(1)}(x'-x'')\,\delta_{\nu_1\nu_2}\,d\,x'\,d\,x'' + \frac{i}{4}\int\!\!\int\{Sp\,[S^{(1)}(x'-x)\,\gamma_\mu\,\bar{S}(x-x')\,\gamma_{\nu_1}]\,\cdot$$
$$\cdot\,[\bar{\psi}^{(0)}(x''),\,\gamma_{\nu_2}\,\psi^{(0)}(x'')]_1 - [\bar{\psi}^{(0)}(x),\,\gamma_\mu\,\bar{S}(x-x')\,\gamma_{\nu_1}\,S^{(1)}(x'-x'')\,\gamma_{\nu_2}\,\psi^{(0)}(x'')]_1 -$$
$$-\,[\bar{\psi}^{(0)}(x''),\,\gamma_{\nu_2}\,S^{(1)}(x''-x)\,\gamma_\mu\,\bar{S}(x-x')\,\gamma_{\nu_1}\,\psi^{(0)}(x')]_1\}\,\delta_{\nu_1\nu_2}\,\bar{D}(x'-x'')\,d\,x'\,d\,x'' -$$
$$-\frac{i}{2}\,\delta\,m^{(2)}\int[\bar{\psi}^{(0)}(x),\,\gamma_\mu\,\bar{S}(x-x')\,\psi^{(0)}(x')]_1\,d\,x'. \qquad (33)$$

Further

$$\frac{i}{2}[\bar{\psi}^{(1)}(x),\,\gamma_\mu\,\psi^{(1)}(x)]_{1,0} = -$$
$$-\frac{i}{4}\int\!\!\int[\bar{\psi}^{(0)}(x''),\,\gamma_{\nu_2}\,\bar{S}(x''-x)\,\gamma_\mu\,\bar{S}(x-x')\,\gamma_{\nu_1}\,\psi^{(0)}(x')]_1\,\delta_{\nu_1\nu_2}\,D^{(1)}(x'-x'')\,d\,x'\,d\,x'' \qquad (34)$$

and we obtain the second order radiative correction to the current operator in the one-particle case as

$$\langle j^{(2)}_\mu(x)\rangle_{1,0} = \frac{i}{2}\int[\bar{\psi}^{(0)}(x),\,\gamma_\mu\,\bar{S}(x-x')\,(\phi(x') - \delta\,m^{(2)}\,\psi^{(0)}(x'))]_1\,d\,x' +$$
$$+\frac{i}{2}\int[(\bar{\phi}(x') - \delta\,m^{(2)}\,\bar{\psi}^{(0)}(x'))\,\bar{S}(x'-x)\,\gamma_\mu,\,\psi^{(0)}(x)]_1\,d\,x' -$$
$$-\frac{i}{4}\int\!\!\int[\bar{\psi}^{(0)}(x'),\,K_\mu(x'-x,\,x-x'')\,\psi^{(0)}(x'')]_1\,d\,x'\,d\,x'' + \text{vac. pol.} \qquad (35)$$

$$\phi(x) = -\tfrac{1}{2}\int\gamma_\lambda\,[S^{(1)}(x-x')\,\bar{D}(x'-x) + \bar{S}(x-x')\,D^{(1)}(x'-x)]\,\gamma_\lambda\,\psi^{(0)}(x')\,d\,x' \qquad (36)$$

$$K_{\mu}(x'-x, x-x'') = \gamma_{\lambda} S^{(1)}(x'-x)\gamma_{\mu}\bar{S}(x-x'')\gamma_{\lambda}\bar{D}(x''-x') + \\ + \gamma_{\lambda}\bar{S}(x'-x)\gamma_{\mu}S^{(1)}(x-x'')\gamma_{\lambda}\bar{D}(x''-x') + \\ + \gamma_{\lambda}\bar{S}(x'-x)\gamma_{\mu}\bar{S}(x-x'')\gamma_{\lambda}D^{(1)}(x''-x'). \qquad (37)$$

These equations are the same as those obtained by SCHWINGER and used by him to compute the anomalous magnetic moment of the electron, the radiative correction to the scattering cross-section etc.[6]

It is not difficult to obtain a similar formula for the radiative correction to the current operator for particles of spin 0. Instead of the factors involving γ-matrices we get expressions of the form (21) and the numerical factors in the formula are slightly changed. In addition, of course, anticommutators must be substituted throughout for the commutators.

V. Conclusions

In the examples treated above the calculations are comparatively simple, when the formalism that has been developed here is used. This is, however, also the case in the original formulation of SCHWINGER. The higher approximations of vacuum polarization in an external field are possibly somewhat simpler here, as we have avoided the ε-functions of SCHWINGER and their rather complicated treatment.[5, 6] This is also achieved in the formalism of DYSON[2] and it remains to compare the merits of the two methods in the higher approximations of the radiative corrections to various operators. DYSON's method, however, does not avoid the complication involved in the normals of the TOMONAGA-SCHWINGER space-like surfaces.

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[6] J. SCHWINGER: Phys. Rev. *76*, 790 (1949).

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Uppsala 1950. Almqvist & Wiksells Boktryckeri AB